Final Report

Course: CIND820

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# 1. Research questions

The research conducted here falls into a literature known as “Growth-at-Risk” (G@R or GaR).

GaR is part of growing and innovative literature that provides an estimate of lost GDP in the future, given a severe stress event (or “tail event”) today. Macroeconomic shocks are occurring more frequently and with greater intensity than even before. A good illustration is the pandemic, where a public health crisis and the associated lockdowns, led to a contraction in economic output that was exceptionally deep and qualifies as a tail event (defined as the 5th quantile of GDP growth distribution.

“Growth-at-Risk” provides an answer to a simple question:

* Given a large shock to the macroeconomy, which is measurable on some forward-looking indicator variable today, can we predict the loss of real GDP growth at the 5th quantile of the growth distribution in the next period?

A loss of real GDP at the 5th quantile of the growth distribution is a maximum measure of GDP contraction. Being able to predicted with sufficient lead-time what such an event would look like using predictive analytics prepares policy makers to implement preventative measures in a timely manner to prevent or significantly mitigate a tail event.

For clarity: while traditional GDP growth forecasting aims to forecast the expected value of GDP growth (defined as the mean of the growth distribution), the goal of GaR is to predict the amount of GDP growth that would be lost if the economy suffered a tail event with potential GDP loss equivalent to the 5th quantile of the growth. For this reason, the GaR approach differs from traditional economic growth forecasting and cannot rely on OLS regression (a point estimate of the expected value). GaR sets the more ambitious task of predicting the entire GDP growth distribution in the future and finding the 5th quantile cut off in the left tail of the distribution. The 5th quantile of the growth distribution is the “growth-at-risk”.

* Traditional economic growth forecasting aims to predict the “expected value” (i.e., the mean of the economic growth distribution
* GaR aims to predict the loss of GDP that would occur in a tail event where growth falls into the 5th quantile of the growth distribution.

Overall, this research falls into the field of “Predictive Analytics”. As part of this project, we will be running 8,769 independent quantile regressions, allowing us to predict the conditional GDP growth distributions for 237 quarters of data. Accordingly, this work falls into the field of “Big Data”.

This work does not fall into the field of machine learning. Quantile regression does not have a “leaning aspect”. We will compare the quantile regression model used here to alternative machine learning models and we conclude that machine learning is not as effective as the quantile regression model used here.

## Contribution compared to past research

This project replicates the GaR framework developed by the International Monetary Fund (IMF). The seminal publication laying out the model for GaR is:

* + Tobias Adrian, “Vulnerable Growth”, Federal Reserve Bank of New York, Staff Report No. 794, September 2016, Revised November 2017

The IMF’s model is being replicated in this project. This project contributes to the research on “Growth-at-Risk” in two ways:

1. It applies the framework to Canada, using Canadian data.
   * + A Canadian application of GaR is currently not available in the literature
   1. The IMF uses a broad financial markets stress as the independent variable
      * This project broadens the field by using total credit as the independent variable.

# 2. The Approach

Growth-at-Risk is a new field of macroeconomics and can be understood by looking at a related literature known as “Value-at-Risk” (VaR).

Value-at-Risk is used in market risk to estimates a maximum trading loss (a loss equivalent to the 5th quantile of the loss distribution).

A graph with a black line

Description automatically generated

Large banks are required by regulators to calculate their daily Value-at-Risk. The idea is simple:

* We are not interested in the mean of the loss distribution (which would represent an average loss)
* We are interested in an estimate of loss that is a reasonable proxy for the maximum loss in the left tail of the loss distribution
* The 5th quantile of the loss distribution is defined as the “Value-at-Risk”:
  + It is a large loss that will at most be exceeded on 5 out of 100 trading days on which losses occur.

Growth-at-risk is in essence identical to Value-at-Risk. Growth-at-risk is not interested in the expected loss of output (the mean of the distribution). Instead, growth-at-risk wants to estimate the maximum loss of GDP that would occur if economic growth fell into the left tail of the GDP growth distribution (represented by the 5th quantile of the growth distribution). The GDP growth that could be lost in this tail event is considered the growth “at risk”.

The approach to growth-at-risk used here can be described in three steps:

1. Future GDP growth distribution (the dependent variable) is estimated using a forward-looking indicator variable (independent variable). For example, the IMF uses a financial stress index (FCI) as indicator variable: if financial conditions are tight today, growth will contract in the next period; if financial conditions are loosened today, growth will expand in the next period.
2. The approach uses quantile regression to estimate the entire distribution of future GDP (in contrast to a point estimate obtained with OLS regression). Specifically, we run quantile regressions from the fifth quantile of the GDP growth distribution to the ninety-fifth quantile, in increments of 2.5 quantiles (for a total of 37 regression estimates per quarter). By smoothing the 37 independent regression estimates, we can predict the full conditional growth distribution (or non-parametric probability density function).
3. As a final step, we fit a skewed Students t-distribution to the empirical, non-parametric PDF by minimizing the squared distance between our estimated, empirical quantiles and the parametric PDF of the skewed t-distribution. This allows us to pinpoint the 5th quantile in the tail of the distribution (labeled as GaR). GaR will shift from quarter to quarter as growth conditions change (orange vs blue growth distribution).

A graph of a graph with arrows pointing to the same line

Description automatically generated

In the approach used here, we estimate quarterly GDP growth distributions conditional on the indicator variable (the flow of credit to the economy in the previous quarter). We go back over the decades to illustrate how the growth distributions change over time.

* This will allow us to visualize the usefulness of the GaR approach
* We have data going back to the 1964. We can calculate quarterly GDP growth distributions for about 60 years (= 237 quarters)
* Because we estimate the full GDP growth distribution at 37 independent quantiles per quarter over 237 quarters, the project makes 8,769 independent calculations (237\*37)

By running 8,769 independent regressions, this project falls into the field of Big Data.

## Description of applied methodology and study design

This project argues that quantile regression is the superior approach to answer the question of this research project: Given a shock (measured by the independent variable) how can we estimate the whole conditional distribution of the dependent variable from tail to tail and find the 5th quantile cutoff?

OLS regression and machine learning approaches cannot answer this question because they estimate the conditional means of the dependent variable (the "expected value") and, as such, are point estimate.

This can be illustrated as follows (see [A. Colin Cameron](https://www.amazon.ca/A-Colin-Cameron/e/B09PWS433K/ref=dp_byline_cont_book_1), [Pravin K. Trivedi](https://www.amazon.ca/Pravin-K-Trivedi/e/B0BDY6NPZX/ref=dp_byline_cont_book_2), *Microeconometrics: Methods and Applications*, 2008):

A graph with blue dots and a red line

Description automatically generated

An OLS regression provides a point estimate of the conditional mean of the distribution The goal of OLS regression is to find the line of best fit that minimizes the sum of the squared residuals between the predicted and actual values of the dependent variable.

Quantile regression represents a more flexible approach for obtaining point estimates at any quantile of the distribution. This is illustrated here:

* An estimate for the 50th quantile (identical to the OLS regression, assuming normally distributed data where mean = median)
* Quantile regression can also provide an estimate at:
  + the 5th quantile of the distribution (where 5% of the observations are below the linear regression line and the remaining observations above)
  + the 95th quantile of the distribution (where 95% of the observations are below the linear regression line and the remaining observations above)

A graph with blue and red dots

Description automatically generated

In this project we will independently fit quantile regression estimates starting at the 5th quantile and proceeding in increments of 2.5 quantiles (5% quantile, 7.5% quantile, 10% quantile, 12.5% quantile . . . ) until we reach the 95% quantile.

* The result is 37 independent regression estimates that allow us to recover the full distribution of the dependent variable.
* By smoothing independent quantile regression estimates with a ridgeline graph, we recover the conditional, non-parametric (empirical) probability density function for the whole distribution (quantiles shown here in increments of five)

A diagram of a function

Description automatically generated

Finally, we can turn the non-parametric (empirical) distribution into a parametric distribution by fitting a skew Student t-distribution to our 37 non-parametric (empirical) quantile estimates.

Smoothing is improved by fitting a skewed t-distribution in order to recover a parametric PDF, described by:

* a mean or central location parameter
* a scale or dispersion parameter
* a fatness of the tails parameter
* a shape parameter

We estimate the four parameters for each quarterly distribution by minimizing the squared distance between our estimated, empirical quantiles and the parametric PDF of the skewed t-distribution.

All these steps have been implemented in the next section (3. The Conducted Analysis).

In section 4, we compare the quantile regression model developed here to other machine learning algorithms.

* We conclude that quantile regression used in this study outperforms machine leaning approaches

# 3. The Conducted Analysis

The data used in this project has been documented in the Abstract as part of Module 1 but is referenced here again for convenience.

* + Quarterly GDP data for Canada -- publicly available from Stats Canada at [The Daily — Gross domestic product, income and expenditure, fourth quarter 2021 (statcan.gc.ca)](https://www150.statcan.gc.ca/n1/daily-quotidien/220301/dq220301a-eng.htm)
  + Total credit to the Canadian economy -- publicly available from the Bank for International Settlements at: [Credit to the non-financial sector (bis.org)](https://www.bis.org/statistics/totcredit.htm)

## The Growth-at-Risk model using quantile regression

Quantile regression requires on two data series – the dependent variable (here: Candain GDP growth) and a forward-looking financial indicator variable. In this project, we use the rate of credit growth as the independent variable to estimate the full conditional distribution of future real GDP growth:

* p(y = GDP growth | x = credit growth).

This economic theory behind this approach is that the growth of total credit is the main driver that determines GDP growth in the next period.

## The data

The BIS data on credit to the Canadian economy contain several credit series as follows:

A graph of different colored lines

Description automatically generated

Specifically, we choose as our indicator variable:

* Credit to private non-financial sector from all sectors

Advantages of choosing this series is that it provides a long history going back to the 1960s. Also, credit to the private sector from all sources is a broad that captures all credit flows (as opposed to credit from banks only) The independent variable chosen looks as follows:

A graph showing growth rate

Description automatically generated

We merge the GDP and credit data into a single data frame. The data frame contains our empirical GDP growth date, our empirical credit growth data, as well lagged variables for GDP growth and credit growth by 1 and 4 periods (which will allow us to predict 1 and 4 periods ahead).

A screenshot of a computer

Description automatically generated

## Specifying the quantile regression model

To estimate GDP growth distribution conditional on the flow of credit, we run a quantile regression and look at in-sample predictions. We run quantile regressions from the fifth quantile to the ninety-fifth quantile, in increments of 2.5 quantiles (for a total of 37 regression estimates per quarter). For clarity, we estimates independently 37 quantiles, covering the full conditional GDP growth distribution.

In contrast to ordinary least squares regression, quantile regression offers several advantages as follows:

* OLS regression estimates the conditional mean of the dependent variable (the "expected value") and, as such, is a point estimate. Quantile regression represents a more flexible approach for modeling the entire conditional distribution of GDP growth from tail to tail. This permits an analysis of possible downside risks, such as tail observations at the fifth quantile of the growth distribution.
* While OLS regression makes parametric assumptions, such as requiring normal distribution of the error terms, quantile regression makes no parametric assumptions. Each of the 37 quantiles in our regression is fitted independently.
* While OLS brakes down given outliers and skew, quantile regression is robust to outliers and highly skewed distributions.

The regression equation used here is:

* eqn.q = Growth ~ Growth\_1 + GCredit\_1.

Note that we are projecting one period ahead by regressing GDP growth on the lagged independent variable.

Note also that we are regressing growth on both itself (lagged) as well as on credit (lagged). Tobias Adrian discusses the rationale for this, explaining the fact that economic conditions are significant in explaining growth in the next period although credit conditions have more explanatory power.

The quantile regression will start at the 5th quantile and proceed increments on 2.5 quantiles until the 95th quantile is reached.

* This is done by setting tau = seq(0.05, 0.95, 0.025)

A screenshot of a computer program

Description automatically generated

The result will be 37 in-sample, empirical (non-parametric) probability density estimates per quarter. Since we have 237 quarters, we run (237\*37) 8,769 independent regression estimates.

* This is an immense computational effort that we document as follows:

A screenshot of a computer

Description automatically generated

## Visualizing the results of 8,769 independent regression estimations

Having used quantile regression to obtain 37 in-sample, empirical (non-parametric) probability density estimates per quarter, we can now proceed to interpolate those quantiles estimates and plot them as ridgeline plots. In essence, we can think of this as simply smoothing the 37 empirical quantile estimates to obtain a non-parametric probability density curve.

The results are shown in the following graph, which shows the conditional GDP growth distributions for each quarter back to the 1964.

* For each quarter, we are able to graph the full conditional distribution of GDP growth
* For each quarter, the whole distribution may shift to the left or the right depending on the amount of credit flowing to the economy
* For each quarter, the shape of the skewed distribution may change – for example, period of recession show GDP distribution with longer left tails that fall into negative GDP growth territory.

## A graph showing a graph of a graph Description automatically generated with medium confidence

The result is an absolutely stunning prediction that the COVID recession would be the deepest recession on the historic record since the early 1960 with GDP growth plunging below -10% in Q2 of 2020. Also the model predicts a spectacular bounce back that is unprecedented in post-World War II history.

The model also predicts that the Global Financial Crisis (2008-09) has the second strongest contraction on record, slightly more than the recessions of 1981-82.

We can also zoom in on the COVID period to get a less crowded look at the massive shifts in GDP growth predicted by the model.

A graph of a graph showing a number of covid-19

Description automatically generated

The above visualizations predict one quarter ahead. However, we can also predict 4 quarter ahead:

* We do this by setting our forecast to 4 quarters (fcast <- 4)

A close-up of a computer code

Description automatically generated

## A graph showing a graph of a wave Description automatically generated with medium confidence

The results are equally compelling. As is generally the case in economics, quarterly data is considerably more volatile than annual data as the year over year (y/y) calculation smooths out the changes that occur in the four intervening quarters. Projecting four quarters ahead, the conditional GDP growth distributions are less volatile. Specifically:

* The conditional GDP growth distributions do not shift as much to the left and the right over time
* the recession signal comes in the form of a longer left tail
* The year ahead projection indicates that there is more variation to the downside (more variation in the left tail than in the right tail). This is a fact that is emphasized in Tobias Adrian’s paper – it’s more likely that shocks to the economy result in lost output and there are few “shocks” that make us better off. This is something that is better visible with the year ahead projection.

At the same time, we note that the results are still broadly comparable. The flow of credit to the economy suggests that the 1980s recession is more severe than the 1970s. The 2009 recession had a greater potential impact than previous recessions and 2020 pandemic lockdown was potentially the worst.

It is important to recall the Tobias Adrian believes that this tool can function as a early warning signal and lead the government to intervene and “prevent” the worst case outcome. The pandemic did see decisive government intervention, explaining the powerful bounce back

## Fitting a skewed student t-distribution and visualizing the fifth quantile cutoff

The final step conducted by Tobias Adrian is to fit a skewed student's t-distribution in order to recover a parametric distribution.

Relative to the t-distribution, the skewed t-distribution adds a shape parameter, which regulates the skewing effect of the probability density function. The skewed t-distribution is described by:

* mean or central location: denoted here as xi
* the scale or dispersion: denoted here as omega
* the fatness of the tails: denoted here as alpha
* a shape parameter: denoted here as tau

The “sn” package in R fits a skewed t-distribution to our empirical quantile estimates by minimizing the squared distance between our estimated, empirical quantiles and the parametric PDF of the skewed t-distribution.

This will allow us to describe and visualize "growth-at-risk" in parametric terms and to label the 5th quantile cut-off.

* It is graphed below as the red dot along with the skewed t-distribution. The 5% quantile in the left tail is the “vulnerable growth” or “growth @ risk”

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A graph of a graph

Description automatically generated with medium confidence

To better illustrate GaR, we can draw a vertical line between the x-axis at the 5% quantile up to the fitted t-distribution PDF. The area to the left of this vertical line under the PDF must accommodate 5% of the probability mass. A short line requires a longer left tail to accommodate 5% of the probability mass; a high vertical line leaves more space between the x-axis and the PDF, forcing the PDF down quickly towards the x-axis and resulting in a shorter tail.

A graph showing a variety of graphs

Description automatically generated with medium confidence

## Evaluating the quantile regression model

We can visualize the change in the quantile coefficients that we empirically estimated. Each black dot is the slope coefficient for the quantile indicated on the x axis. The red lines are the least squares estimate and its confidence interval.

* Clearly, the lower and upper quartiles are well beyond the least squares estimate of the mean of the distribution – an indication that OLS is inadequate to capture the full conditional distribution.

A graph with lines and numbers

Description automatically generated

For both the independent variables Growth\_1 and GCredit\_1, the linear regression slope at each quantile is sufficient to describe the relationship between x and y.

* For Growth\_1, only two dots fall outside of the 95% percent confidence interval
* For GCredit\_1, only two dots fall outside of the 95% percent confidence interval

A graph of a graph

Description automatically generated

A graph of a graph

Description automatically generated

# 4. Other Models including Machine Learning

We could rhetorically pose the key question:

* Do machine learning algorithm improve on the quantile regression model presented above?

The answer is a resounding “no”. There is no machine learning algorithm that can estimate the whole conditional growth distribution of the dependent variable using a forward looking independent indicator variable.

Machine learning builds on regression analysis to offer algorithms to make forward-looking predictions. Unfortunately, those algorithms all fall into the class of OLS or regression trees. Such algorithms aim to predict the “expected value” – however, they completely useless to estimate tail observations at the fifth quantile of the distribution

We illustrate with several examples:

1. Single regression:

We regressed our GDP data on time to generate a trend prediction. We divided the empirical data into train and test sets. We illustrate the results here the algorithm accurately fits a trend line to test data, which is compatible with the learning from the train data. We visualize as follows and point to the dotted forecast based on the training set.

A graph with a red line

Description automatically generated

Although the algorithm was able to learn the trend and project accurate ahead, this is just an OLS point estimate of expected value.

* This is a problem from the point of view of wanting predict the entire conditional growth distribution based on 37 independent quantile estimates (instead of the “expected value”)

We also perform a residual analysis to evaluate the effectiveness of the model:

A graph and diagram of a graph

Description automatically generated with medium confidence

The time plot should show a random pattern but shows that the error terms are serially correlated. This is confirmed by the autocorrelation function, showing statistically significant autocorrelation that decays only gradually over time. The histogram is significantly skewed and the error terms are not normally distributed.

We conclude that the model is inadequate.

1. Multiple regression algorithm:

Another machine learning algorithm builds on multiple OLS regression and we regress growth on the two independent variable we also used in the quantile regression – growth lagged by one period and credit lagged by one period. We visualize the results as follows:

A graph showing a graph

Description automatically generated

The fitted values generated by the algorithm project one period ahead are remarkably accurate when compared to the empirical growth data.

* But this is again an OLS regression that forecasts the “expected value”. This is a problem from the point of view of our goal of wanting to predict the entire distribution

We also perform a residual analysis to evaluate the effectiveness of the model:

A screenshot of a graph

Description automatically generated

The residual analysis shows appropriate normality of error terms. However, the time changing variation over time, especially as we head into the COVID period. The autocorrelation plot show a statistically significant spike at lag 4, which suggests seasonal correlation of our quarterly data. The histogram shows that the error terms are approximately normally distributed.

Overall, this is a far better model than the previous one. The main downside is that it cannot estimate the whole conditional distribution of the dependent variable.

1. Regression trees

We explore regression trees but are skeptical whether decision trees are better suited for this task. Generally, decision trees are preferable if the independent variables include many features or many complex, nonlinear relationships among features and the outcome; in this case, we only have two independent variables.

Here we use the same data and ask if regression trees can improve on the single and multiple OLS regression. We split the data into train and test sets to be able to judge the performance of the model. Based on the train data, the algorithm splits the data as follows:

A diagram of a graph

Description automatically generated

We note that, in numeric decision trees, homogeneity is measured by statistics such as variance, standard deviation, or absolute deviation from the mean (as opposed to classification trees, where homogeneity is measured by entropy).

We note that the algorithm does do a reasonable job in predicting the median and mean values of the distribution.

* These predictions, based on the training set, align somewhat with the actual expected valued of the test set.

However, a quick look at the summary statistics reveals a very a large potential problem: the predictions fall on a much narrower range than the true values.

* The algorithms predicts the following range from Min to Maz:

A screenshot of a computer

Description automatically generated

* The actual values of the test data are much more dispersed:

A screenshot of a computer

Description automatically generated

This is again a very large problem considering our interest in the tails of the distribution.

We conclude that regression trees are better equipped to estimate the mean of a distribution. They are nor very helpful given our interest in the tails of the distribution.

# 5. List of Findings

We record the first key finding of this project as follows:

1. Quantile regression allows us to take the flow of total credit to the Canadian economy and accurately predict the **whole conditional GDP growth distribution** in the next quarter!
   * 1. We did this accurately for many decades and were able to see that the model forecast a much deeper contraction during COVID then ever happened previously
     2. We also saw that quantile regression accurately predicted and unprecedented bounce back.

We also note the second most important conclusion:

1. We reject all regression-based machine learning algorithms as incapable of predicting the conditional growth distribution
   * 1. Regression-based machine learning algorithms generally succeed in predicting the expected value by making a point estimate of the mean of the distribution
     2. Regression-based machine learning algorithms do not help us estimating tail events

The quantile regression conducted here shows significant differences if we project one quarter or one year ahead. We conclude:

1. As expected, quarterly data is more volatile than annual data, given that annual data averages out the variations that occur on a quarterly basis.
   * 1. Quarterly data has more signalling power and should be preferred

This project applied the IMF’s existing concept of Growth-at-Risk to the Canadian economy and, instead of using the IMF’s broad financial stress index, we used total credit as the indicator variable. We conclude:

1. This project broadens and confirms the usefulness of the IMF’s GaR model in the case of Canada
2. This project broadened the IMF’s work by using total credit as the indicator variable

## Shortcomings of the work and concluding remarks

The implementation of GaR as presented here uses a single credit indicator as the explanatory variable to estimate the quantiles of the conditional GDP growth distribution p(y = GDP growth | x = credit growth). The major difference with the IMF’s implementation of Tobias Adrian’s model is that the latter is based on a wider financial conditions index. This difference has important implications, specifically:

* Amount of left skew: Tobias Adrian’s conditional GDP distributions are much more left skewed when compared to the ones presented here.
* Stable right tails: Adrian emphasizes the stability of the right tails in his implementation and notes that downside risk in the left-tail varies much more strongly over time than upside risk. By contrast, the conditional growth distributions estimated here with credit growth are more symmetrical and show considerable upside risk, shaped presumably by the enormous credit expansions we continue to see. This more symmetric up and downside risk may provide valuable insights if we take a Minskian approach that emphasizes excessive credit creation over the cyclical upturn as the inevitable root of the following credit contraction.
* Shifts in the conditional GDP growth distribution: The quarterly, conditional GDP growth estimated by credit keep their symmetrical shape over time but show bigger shifts to both the left and the right over time (when compared to Adrian et al, where much of the movement over time occurs as the left tails wax and wane).

Overall, these differences are not necessarily shortcomings. They contribute a broadening of the research on the subject of “Growth-at-Risk.”

# 6. References and Github

Growth-at-risk is a relatively recent filed but it is already reasonably well documented and consistently used by the IMF. Important publications that provide a full conceptual understanding of the idea and approach include:

1. Tobias Adrian, et al., “Vulnerable Growth”, Federal Reserve Bank of New York, Staff Report No. 794, September 2016, Revised November 2017
   * Economic forecasts usually provide point estimates for the conditional mean of GDP growth
   * GaR models the full distribution of future real GDP growth as a function of current financial and economic conditions using quantile regressions
   * Once empirical estimates of the quantiles have been obtained, a distribution can be drawn by interpolating between the estimated quantiles. In addition, a skewed t-distribution (a flexible distribution function with four parameters) could be fitted to the quantile estimates. This allows the GaR model to transform the empirical quantile distribution into an estimated conditional distribution of GDP growth.
     + The procedure is computationally straightforward and transforms the inverse cumulative distribution function from the quantile regression into a density function.
2. George Cooper, “The Origin of Financial Crises: Central Banks, Credit Bubbles and the Efficient Market Fallacy”, 2012
   * Argues that the flow of credit gives rise to the boom-bust cycle
     + This is the foundation of the approach adopted in this project
     + We will use the flow of credit as the forward-looking indicator variable to estimate future distributions of GDP growth
3. Ananthakrishnan Prasad, et al., IMF Working Paper, “Growth at Risk: Concept and Application in IMF Country Surveillance”, February 2019
   * Explains how the IMF uses GaR for tracking the evolution of financial conditions and macro-financial vulnerabilities
   * This can provide valuable information for policymakers regarding risks to future growth and, hence form a basis for targeted pre-emptive action
   * GaR analysis can appreciably expand the macrofinancial surveillance toolkits of policymakers
   * The GaR approach has been incorporated into the International Monetary Fund’s macro-financial surveillance toolkit. In the context of multilateral surveillance, the Global Financial Stability Report (GFSR) has explored tail risk to global economic growth based on prevailing global financial conditions.
4. IMF, “Global Financial Stability Report” October 2017
   * Chapter 3, “Financial Conditions and Growth-at-Risk” provides a very accessible overview of the GaR model adopted by the IMF
   * Argues that financial conditions predict increased downside risks to GDP growth in most advanced economies and a more uncertain growth outlook in emerging markets.
5. Tobias Adrian, et al., “The Term Structure of Growth-at-Risk,” IMF Working Paper 2018/180.
   * Extends the GaR model into the future by modeling a “term structure” of growth-at-risk.
   * The main idea is that policy makers may reduce the probability of systemic crisis today by intervening with expansionary credit and spending policies to support growth, but this may also increase the future amount of growth that is “at risk”.
6. [Nellie Liang](https://www.brookings.edu/people/nellie-liang/) and [Tobias Adrian](http://www.tobiasadrian.com/), “How Growth-at-Risk can help central bankers gauge financial stability risks” Brookings, April 11, 2019).
   * Summary of the idea of a “term structure” of growth-at-risk
   * When asset prices fall and financial market volatility rises, monetary policymakers face a dilemma.  An interest rate cut would reduce downside risks to the economy and support economic growth.  But it could also fuel risk taking, leading to higher asset valuations, more leverage, and other buildups of financial vulnerabilities, raising future risks
7. Kevin Doyd, “Measuring Market Risk” 2nd Ed, 2005
   * Chapter 3 provides an overview to “Measuring Value-at-Risk”, which can be done with historical or parametric approaches
   * The concept of Value-at-Risk is identical to the concept of Grwoth-at-Risk. Value-at-Risk provides a more established literature that may be useful to understand the foundation of the idea of Growth-at-Risk
8. A. Colin Cameron and Pravin K. Trivedi, “Microeconometrics: Methods and Applications”, 2nd Edition, 2005
   * Good coverage of quantile regression

## Github Repository

This project is available at:

https://github.com/CJRitschl/TMU2/tree/main